Facial incidence colourings of embedded multigraphs

Mirko Horňák[(1)], Stanislav Jendrol'[(1)], Roman Soták[(1)]

[(1)] P.J. Šafárik University, Košice, Slovakia

Let G be a cellular embedding of a multigraph in a closed surface. Edges $e_1, e_2 \in E(G)$, $e_1 \neq e_2$, are facially adjacent if there exists a face $f \in F(G)$ such that e_2 follows immediately after e_1 in some facial walk of f. An incidence of G is a pair (v, e), where $v \in V(G)$, $e \in E(G)$ and $v \sim e$ (v is incident with e). Two distinct incidences (v_1, e_1) and (v_2, e_2) of G are *facially adjacent* if either (a) $e_1 = e_2$ or (b) e_1, e_2 are facially adjacent and either (ba) $v_1 = v_2$ or (bb) $v_1 \neq v_2$ and there is $i \in \{1, 2\}$ such that $v_j \sim e_i$, $j = 1, 2$. A *facial incidence colouring* of G assigns a colour to each incidence of G in such a way that facially adjacent incidences get distinct colours.

As usual, the minimum number of colours in a facial incidence colouring of G is studied, and that minimum is proved to be always at most 7. For several families of embedded graphs better bounds are presented.