

# Graph Products and Symmetry Breaking in Graphs

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A coloring of the vertices of a graph  $G$  is said to break the symmetries of  $G$  if it is only preserved by the trivial automorphism. The minimum number of colors of such a coloring is the distinguishing number of  $G$ .

Despite the fact that the structure of the automorphism group of graph products is well known, there are still open problems about their distinguishing numbers. Many of them pertain to weak products of infinitely many factors. We address such problems for the Cartesian, the strong, and the direct product.

On the way we investigate properties of weak products and describe their role in the characterization of median graphs with non-exponential growth and in the game of cops and robbers.

Then we turn to countable graphs  $G$  with distinguishing 2-colorings where one color class is finite. The minimum number of elements in the finite class of all such colorings is the cost  $\rho(G)$  of 2-distinguishing  $G$ .

We characterize such graphs and give bounds on  $\rho(G)$ . The best bounds are for two-ended graphs and depend on their product-like properties.